

# ANALYSIS OF MICROSTRIP DISCONTINUITIES USING THE FINITE DIFFERENCE TIME DOMAIN TECHNIQUE

C. J. Railton and J. P. McGeehan

Centre for Communications Research, Faculty of Engineering,  
University of Bristol, BRISTOL, BS8 1TR, UK

## ABSTRACT

This contribution demonstrates the potential of the Finite Difference Time Domain technique to analyse MMIC structures of arbitrary complexity with moderate computational effort and to meet the requirement for CAD tools capable of treating high density MMIC's. Results are presented for uniform microstrip, the abrupt termination and the microstrip right angle bend.

## INTRODUCTION

The current movement in Microwave Monolithic Integrated Circuits (MMIC's) is towards higher operating frequencies, higher component densities and novel components of increasing complexity. This trend is made possible by advances in fabrication technology. Unfortunately, the available CAD tools for analysing such compact, high frequency circuits are inadequate for this task and in order to fully exploit the fabrication techniques which are available, it is necessary that the state of the art in CAD also advance.

The method of analysing MMIC components, which is utilised in the most advanced CAD package, is the Spectral Domain Method (SDM) [1]. This method owes its success largely to the fact that the asymptotic behaviour of the fields in the region of a metal edge are known *a priori*. However, as the structures to be analysed become more complex, and the interactions between components become more significant, the SDM loses its advantages and a method of more general applicability is required.

The Finite Difference Time Domain (FDTD) technique has been much used in the analysis of the scattering of electromagnetic radiation from objects of complex shape [2,3]. More recently it has been applied to the analysis of planar transmission lines such as microstrip and finline [4].

It is the purpose of this contribution to demonstrate the potential of the FDTD technique in the rigorous full wave analysis of MMIC structures and to show some ways in which the basic technique may be improved to increase its versatility and efficiency. Results are presented for the propagation constant of uniform microstrip, calculated using this method, which show excellent agreement with those obtained using SDM. Results are also presented for the parameters of structures containing a right angle bend in microstrip and an abrupt termination in microstrip. The latter results are compared to those obtained in [5] by means of the SDM where good agreement can be seen. Thus it is shown that the FDTD method has great potential for the analysis of MMIC components of arbitrary complexity. Work is currently in progress to apply this technique to the microstrip step and T junction discontinuities and to include the effects of finite strip thickness and conductivity.

## APPLICATION TO UNIFORM MICROSTRIP

In order to demonstrate the validity of the method, it was first applied to a uniform microstrip whose geometry is shown in Fig. 1. Metal walls are placed across the microstrip forming a resonator in the manner of the problem analysed in reference [5]. The box was divided up into regions of different sizes, each of which contain a fixed number of nodes. By this means, a greater density of nodes can be placed in the regions of greater field variations. Since the asymptotic behaviour of the field at the edge of the strips is known, the necessity for a high node density near the edges of the metal can be mitigated by using a finite difference formula which takes this into account. An example of this is described later. The initial field pattern was chosen to be similar to that which would exist in the microstrip in the steady state. This choice of excitation causes the subsequent analysis to be stable and to converge quickly to the steady state. By these means the available *a priori* information is utilised to speed up the computation.

In order to find the exact resonant frequencies of the structure, one must Fourier Transform the time sequence. Unfortunately, the Fast Fourier Transform algorithm is not adequate since it only provides the transform at discrete frequencies. It is, therefore, necessary to calculate the transform directly so that the value at any frequency can be ascertained. The positions of the maxima of the transform can then be found using standard methods. If the transform is performed on the raw time sequence, then as well as the desired peak, there will be sidelobes of considerable amplitude. These reduce the accuracy of the result and make it difficult to apply an algorithm which automatically finds the position of the maximum. This problem has been overcome by multiplying the time sequence with a windowing function. The effect of this can be seen in Fig 2. which shows the convergence of the result as more iterations are taken. Fig 2a. is with a raised cosine window, Fig 2b. is without windowing. The improvement gained by using the windowing can clearly be seen.

Using these techniques, the theoretical results tabulated in Table 1 have been obtained. It can be seen that using the dominant resonant mode the agreement with the SDM is within 0.25%. Accuracy is less when the higher order modes are used since the mesh size, relative to a wavelength, is larger.

#### INCORPORATION OF EDGE SINGULARITY

Consider the structure in Fig. 3. Here we have a metal edge whose boundary is somewhere between the  $E_z$  node and the  $H_y$  node. In order to calculate the value of  $H_y$  the standard finite difference formula would include  $(E_{z2} - E_{z1})/\delta x$  which assumes that  $E_z(x)$  varies linearly in the region between the nodes. In fact it is well known that, close to the edge,  $E_z(x) = k\sqrt{r}$  where  $r$  is the distance from the edge and  $k$  is a constant. We can then say:

$$E_{z1} = k \sqrt{\delta x - \delta} \Rightarrow E_z(x) = E_{z1} \sqrt{r} / \sqrt{\delta x - \delta}$$

$$\partial E_z / \partial x = E_{z1} / 2\sqrt{r(\delta x - \delta)}$$

Now we require  $\partial E_z / \partial x$  at the  $H_y$  node, ie.  $r = \delta x/2 - \delta$ . Thus:

$$\left. \frac{\partial E_z}{\partial x} \right|_{r=\delta x/2 - \delta} = E_{z1} / 2 \sqrt{(\delta x/2 - \delta)(\delta x - \delta)}$$

We may replace the  $E_{z2} - E_{z1}$  in the standard formula with the above expression which will give a more accurate answer.

This method of incorporating the known asymptotic field behaviour into the finite difference algorithm can also be applied to more complex shapes such as bends, corners or strips having finite thickness.

It is interesting to note that, in the case of a uniform microstrip, if  $\delta/\delta x = (1+\sqrt{5})/4$  then the answer given by the two formulae will be the same. This value corresponds to the edge being somewhat less than half way between the  $E_z$  node and the  $H_y$  node.

#### APPLICATION TO THE ABRUPT TERMINATION

In Fig. 4. we have an enclosed resonator containing a microstrip line which terminates abruptly. We can apply the FDTD method in the same way as for uniform microstrip, taking into account the known behaviour of the field at the  $90^\circ$  corners, in order to calculate the resonant frequencies of the structure. We can calculate the propagation constants at these frequencies using the SDM thence ascertain the effective length extension of the termination. By performing the calculation for strips of different lengths, the dependance of the effective extension on frequency can be found.

In Table 2 the results obtained for the abrupt termination are compared to those given in [5]. It can again be seen that good agreement exists.

#### APPLICATION TO THE RIGHT ANGLE BEND

Fig. 5 shows the geometry of a microstrip line containing a  $90^\circ$  bend. In order to analyse this structure, we again place the structure under test in a closed box and choose the lengths  $l_1$  and  $l_2$  to provide resonant frequencies in the region of interest. Since we are now dealing with a two-port structure, there are three unknown parameters which must be determined for its complete characterisation. Thus it is necessary to find three combinations of  $l_1$  and  $l_2$  which will yield the same resonant frequency. In practice, the reflection coefficient is very low and the phase of the transmission coefficient can be calculated using just one geometry.

Table 3 shows the calculated effective length of the strip plotted against frequency. Fig 6 shows isometric plots of the calculated  $x$  and  $z$  components of the electric field in the plane containing the metallisation for the geometry of Fig. 5. The expected edge singularities and modal field patterns can clearly be seen.

## CONCLUSION

In this contribution it has been demonstrated that the FDTD method has much potential in the area of the analysis and CAD of complex MMIC structures and is capable of being enhanced in a number of ways to improve computational efficiency and accuracy.

## References

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3. A. Taflove and K. Umashankar "Radar Cross Section of General Three Dimensional Scatterers" IEEE Trans EMC-25 November 1983 pp 433 - 440.
4. D.H. Choi and W.J.R. Hoefer "A Graded Mesh FD-TD Algorithm for Eigenvalue Problems" Proc. EuMC 1987 Rome pp 413-418
5. R.H. Jansen "Hybrid mode analysis of end effects of planar microwave and millimetrewave transmission lines" Proc IEE Vol 128 Pt. H No. 2 April 1981. pp 77-86.

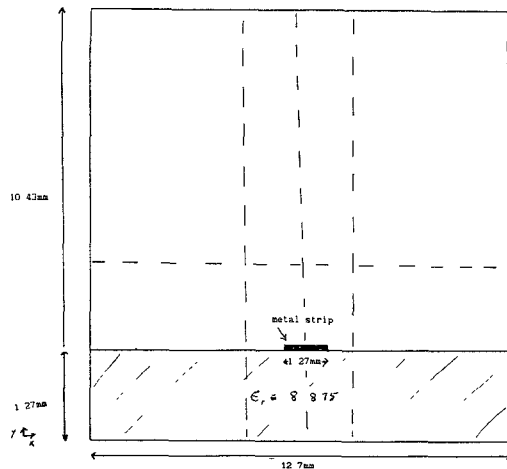


Fig 1 - Geometry of uniform microstrip  
Dashed lines show the division into regions in the x-y plane

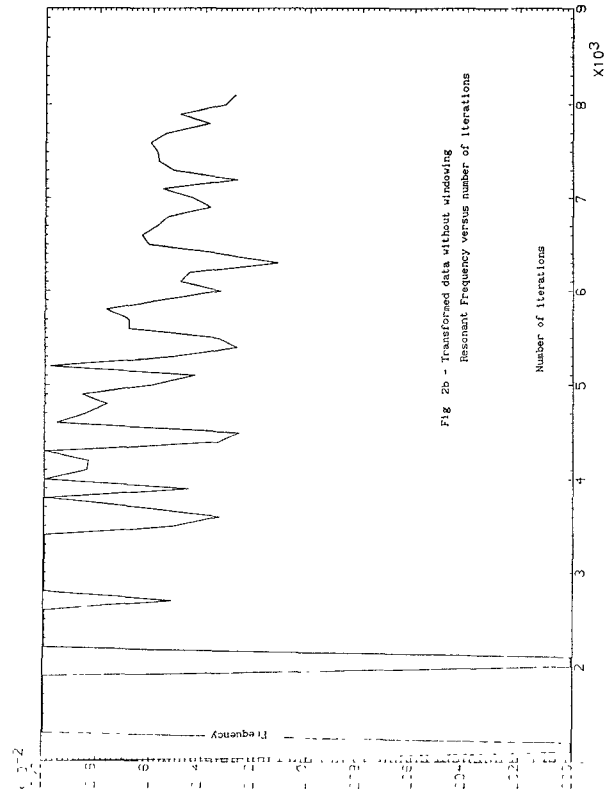


Fig 2b - Transformed data without windowing  
Resonant Frequency versus number of iterations

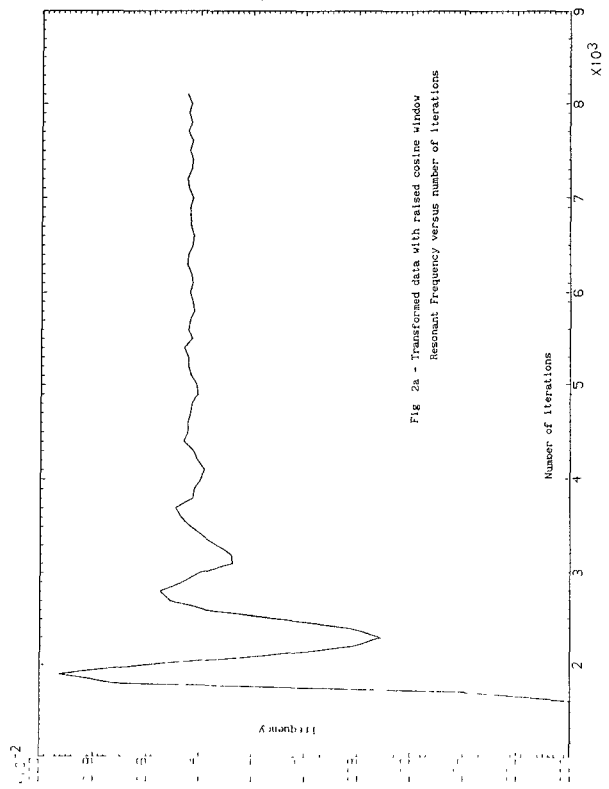


Fig 2a - Transformed data with raised cosine window  
Resonant Frequency versus number of iterations

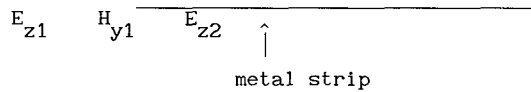


Fig. 3 Nodes close to the edge of a metal strip

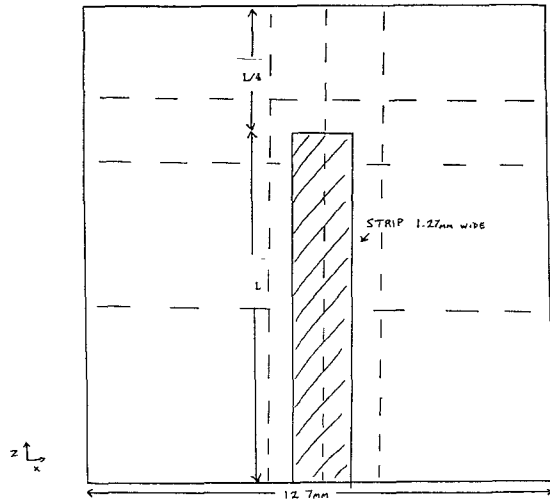


Fig. 4 - Geometry of a microstrip abrupt termination. Dashed lines show the division into regions in the x-z plane

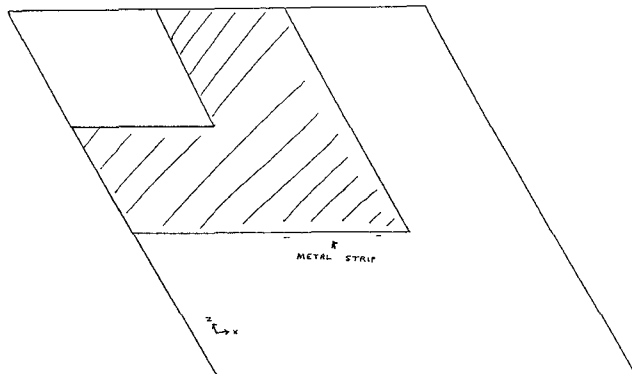


Fig. 5 - Geometry of a microstrip right angle bend  
Strip width = 3.81mm  $\epsilon_r = 10$

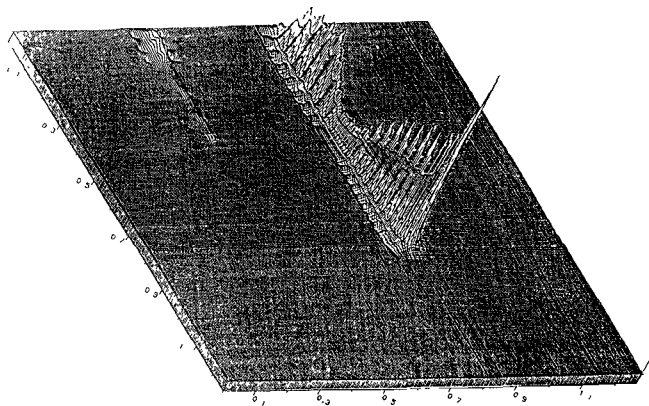


Fig. 6a - Plot of intensity of  $E_x$  in the plane of the strip  
for the geometry of Fig. 5, strip length = 12mm.

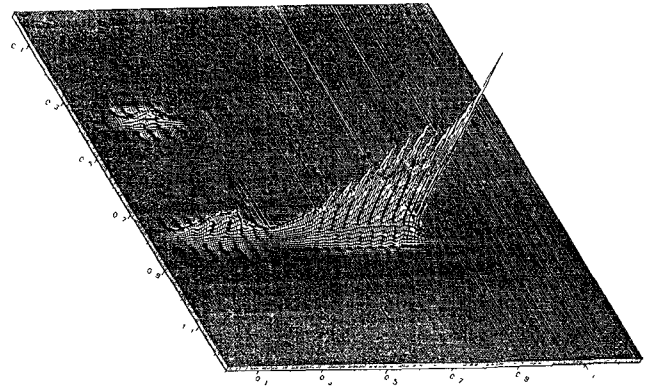


Fig. 6b - Plot of intensity of  $E_z$  in the plane of the strip  
for the geometry of Fig. 5, strip length = 12mm

Table 1- Results for a uniform microstrip

Length of box (mm)	resonant frequency GHz	$\epsilon_{eff}$ from FDTD	$\epsilon_{eff}$ from SDM	% difference
20	3.052	6.04	6.05	0.18
18	3.383	6.07	6.08	0.16
16	3.787	6.13	6.11	0.25
14	4.316	6.16	6.16	0.00
20	5.952	6.35	6.31	0.63
18	6.582	6.41	6.37	0.66
16	7.363	6.48	6.44	0.69
14	8.359	6.57	6.53	0.70
20	11.393	6.93	6.77	2.20
18	12.574	7.02	6.86	2.30
16	13.952	7.23	6.96	3.70
14	15.891	7.28	7.10	2.54

Table 2 - Results for microstrip abrupt termination

Length of strip (mm)	resonant frequency GHz	$\epsilon_{eff}$ SDM	$\delta l/t$ FDTD	$\delta l/t$ [5]	difference
15	1.988	5.971	0.34	0.32	0.02
10	2.927	6.040	0.33	0.32	0.01
8	3.610	6.098	0.32	0.32	0.00
6	4.703	6.197	0.31	0.30	0.01
15	5.811	6.289	0.31	0.30	0.01
4	6.700	6.380	0.34	0.30	0.04
10	8.429	6.532	0.33	0.29	0.04

Table 3- Results for the geometry of Fig. 5  
Strip width = 3.81mm

Length of strip (mm)	resonant frequency GHz	$\epsilon_{eff}$ from SDM	Effective Length of strip (mm)
40	1.364	7.37	40.51
30	1.809	7.43	30.43
20	2.655	7.56	20.55
16	3.270	7.66	16.57
12	4.192	7.83	12.79
8	5.676	8.07	9.3